Due: 6th of May 2010 at 10am

Please submit as PDF by email to `thomas.wolle@nicta.com.au`

As a first step go to `http://gudmundsson.biz/comp5045/course_information.htm` and read the sections: **Advice on how to do the home work** and **Academic Dishonesty**.

COMP 5045 – Assignment 2

1. **(see book exercise 7.3)** Show that $\Omega(n \log n)$ is a lower bound for computing Voronoi diagrams by reducing the sorting problem to the problem of computing Voronoi diagrams. You can assume that the Voronoi diagram algorithm should be able to compute for every vertex of the Voronoi diagram its incident edges in cyclic order around the vertex. [10 points]

2. **(Point queries among rectangles)** Let P be a set of n rectangles in the plane. Report all rectangles in P that intersect a query point q. Describe a data structure for this problem that uses $O(n \log n)$ preprocessing, $O(n)$ storage and $O(n^{3/4} + k)$ query time. (*Hint:* How can a rectangle in 2D be described in 4D? How can a rectangle in 1D be described in 2D?) [10 points]

3. **(see book exercise 5.3)** In the lecture it was indicated that kd-trees can also be used to store sets of points in higher-dimensional space. Let P be a set of n points in d-dimensional space. In parts (a) and (b) you may consider d to be a constant.

   (a) Describe an algorithm to construct a d-dimensional kd-tree for the points in P. Prove that the tree uses linear storage and that your algorithm takes $O(n \log n)$ time. [2 points]

   (b) Describe the query algorithm for performing a d-dimensional range query. Prove that the query time is bounded by $O(n^{1-1/d} + k)$ (where k is the size of the output). [3 points]

   (c) Show that the dependence on d in the amount of storage is linear, that is, show that the amount of storage is $O(dn)$ if we do not consider d to be a constant. Give the dependence on d of the construction time and the query time as well. [5 points]

4. **(see book exercise 2.14)** Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that p can see, that is, all line segments of S that contain some point q so that the open segment $\overline{pq}$ does not intersect any line segment of S. Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p. [10 points]

5. The maximum degree of a graph is the maximum number of edges incident to a vertex in the graph.

   (a) Prove that the degree of a Nearest Neighbour (NN) graph of a point set in the plane is at most 6. [5 points]

   (b) Prove that the degree of a Euclidean minimum weight spanning tree (MST) of a point set in the plane is at most 6. [5 points]

6. Consider a set S of n points in the plane. Each point represents a city and contains its name, its two coordinates and its population. The aim is to preprocess S, such that the following type of queries can be answered effciently: given a axis-parallel rectangle R, return the city with the largest population within R. For full points give a data structure using $O(n \log n)$ space and preprocessing time and $O(\log^2 n)$ query time. [10 points]